Correspondence

Analysis of a Two-Section Coupler*

The use of single-section quarter wavelength TEM mode directional couplers having excellent wide-band performance is well known.[1] A method for decreasing the coupling variation with frequency by cascading three quarter wavelength couplers has also been described.[2] The purpose of this note is to analyze, for loose coupling, the behavior of a two quarter wave section coupler and compare the results with a coupler consisting of three sections.

The basic coupler configuration is shown in Fig. 1. For an incident voltage of unity, the coupled voltage V_c can be written as (neglecting the time dependence)

$$V_c = j\beta \int k(x)e^{-j2\beta x}, \quad (1)$$

where $|k(x)|^2 \ll 1$

$$V_c = j\beta\left[\int_0^{L_1} k_1 e^{-j2\beta x}dx + \int_0^{L_2} k_2 e^{-j2\beta x}dx\right]. \quad (2)$$

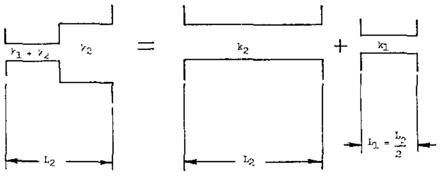

Fig. 1—Superposition of two single-section couplers to form a two-section coupler.

Upon integrating and using the fact that $L_2 = 2L_1$, we obtain

$$V_c = je^{-j\beta L_1}[k_1 \sin \beta L_1 + e^{-j\beta L_1}k_2 \sin 2\beta L_1]. \quad (3)$$

By letting $k_2 = 0$, the familiar result for the single-section quarter wave coupler is obtained.

One can note from (3) that the phase shift of the coupled wave is no longer 90°, as it would be for the single-section or three-section couplers.

The quantity of interest is $|V_c|^2$. Expanding the portion of (3) in the brackets and squaring the real and imaginary parts, we have

$$|V_c|^2 = k_1^2 \sin^2\theta + 2k_1k_2 \sin\theta \cos\theta \sin 2\theta + k_2^2 \cos^2\theta \sin^2 2\theta + k_2^2 \sin^2\theta \sin^2 2\theta, \quad (4)$$

where $\theta = \beta L_1$.

Combining terms

$$|V_c|^2 = \sin^2\theta[k_1^2 + 4k_1k_2 \cos^2\theta + 4k_2^2 \cos^2\theta] \quad (5)$$

or

$$|V_c|^2 = k_1^2 \sin^2\theta[1 + 4r(1 + r) \cos^2\theta], \quad (6)$$

where $r = k_2/k_1$.

In Fig. 2, the bandwidth ratio of the two-section coupler is plotted as a function of the coupling variation for an equal ripple response. A comparison with the three-section coupler shows that, for practical purposes, one does not buy much in the way of improved performance over a two-section coupler by going to the extra section. For example, the coupling variations for a three-to-one bandwidth are 0.21 and 0.26 db respectively, or a difference of only 0.05 db. However, both three- and two-section couplers exhibit substantially better performance than a single-section coupler. Fig. 3 gives the coupling variation as a function of r.

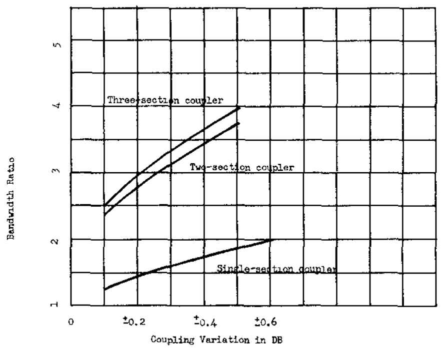

Fig. 2—Comparison of couplers.

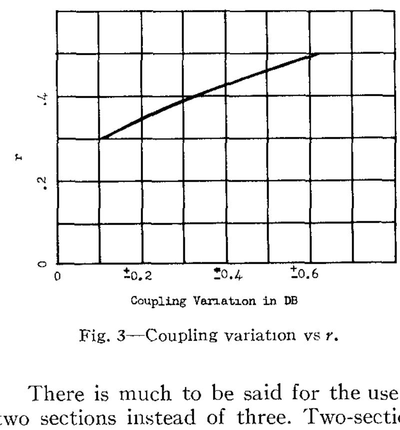

Fig. 3—Coupling variation vs r.

There is much to be said for the use of two sections instead of three. Two-section couplers are shorter, have less discontinuities and so have better VSWR and directivity specifications, and cost less to manufacture.

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* Received February 14, 1962.

[1] B. M. Oliver, "Directional electromagnetic coupling," Proc. IRE, vol. 42, pp. 1686–1692; November, 1954.

[2] J. K. Shimizu and E. M. T. Jones, "Coupled-transmission-line directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-6, pp. 403–410; October, 1958.

Measuring the ω-β Diagram of Periodic Structures*

One of the most important properties of a periodic structure is the shape of the ω-β diagram. A common technique for measuring the ω-β diagram is to form a resonant section by placing shorting planes at positions of symmetry within the periodic structure and to observe the resonant frequencies of the resulting resonator.[1] Discussed in this correspondence is a technique wherein the far end of the periodic structure is shorted and the positions of nulls of voltage on an input line are observed as frequency is varied. From these nulls it is possible to determine the frequencies where the electrical length of the loaded structure is a multiple of π radians. Basically, it is a procedure for graphically determining points on the ω-β diagram.

As an example of the technique, consider a periodic structure formed by a TEM line with six loading elements to give five sections as shown in Fig. 1. The unloaded

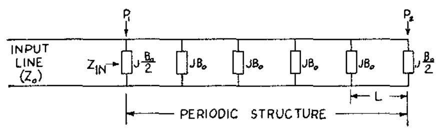

Fig. 1—Uniform periodic structure with five sections.

characteristic impedance of the line used in the periodic structure is assumed to be the same as that of the input line. Following the work of Slater,[2] the equation for the phase constant β_0 per section of the periodic structure is

$$\cos\beta_0 L = \cos\beta L - \frac{B_0 Z_0}{2}\sin\beta L \quad (1)$$

where β is the phase constant per length L of the unloaded line and Z_0 is the unloaded characteristic impedance. Assuming capacitive loading, for example, $B_0 = \omega C_0$ and (1) becomes

$$\cos\beta_0 L = \cos\beta L = \beta L(K_1) \sin\beta L, \quad (2)$$

where

$$K_1 = \frac{C_0 Z_0}{2L\sqrt{\mu_0\epsilon_0}}. \quad (3)$$

Continuing with the example and taking

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[1] D. A. Watkins, "Topics in Electromagnetic Theory," John Wiley and Sons, Inc., New York, N. Y., pp. 9–10; 1958.

[2] J. C. Slater, "Microwave Electronics," D. Van Nostrand Co., Inc., New York, N. Y., pp. 177–186; 1950.